

Engineering Mathematics and Physics Department Algebra and Analytical Geometry Course Code: Math 102 Final Exam Time Allowed: 2 hours	 <b>Modern University</b> For Technology & Information <b>Faculty of Engineering</b>	Academic year: 2009/2010 Semester: Spring May, 16, 2010 Examiners: Dr. Mona Mehanna Dr. Mohamed Husein Eid
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**Answer Five questions only**

**Question 1**

- (a) Find the vertex, focus, and sketch the parabola  $y^2 - 4x - 4y = -20$ .
- (b) Find the center, vertices and foci of the curve  $x^2 + 4y^2 - 8x - 16y + 16 = 0$ .

**Question 2**

- (a) Separate the lines and find the angle between them  $x^2 - y^2 + 3x - y + 2 = 0$ .
- (b) Write the equation of the plane which passes through the point (1, -1, 3) and parallels to the plane  $2x + 3y - z + 12 = 0$ .

**Question 3**

- (a) Write the equation of hyperbola of foci  $F_1(5, 0)$ ,  $F_2(-1, 0)$  and transverse axes is 4.
- (b) Describe each of the following surfaces: (i)  $x^2 - y^2 + z^2 = 0$   
 (ii)  $x^2 + y^2 + z^2 - 2x - 8z + 6 = 0$       (iii)  $x^2 + y^2 - 2z = 0$
- (c) Determine the centre and radius of the circle  $x^2 + y^2 - 2x + 4y - 4 = 0$ . Also, write it in parametric form.

**Question 4**

- (a) Solve the equation  $x^3 - 5x^2 + 8x - 6 = 0$ , if one of the roots is (1+ i).
- (b) Using mathematical induction to prove the validity of the following:  
 $1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{1}{6}n(n + 1)(2n + 1)$ .

**Question 5**

- (a) Using the binomial theorem, expand  $(8 + 12x)^{\frac{2}{3}}$ .
- (b) Use De Moiver's theorem to evaluate:  $(-\sqrt{3} + i)^{\frac{3}{5}}$ .
- (c) Find the sum  $\sum_{r=1}^n \frac{1}{(r+1)(r+3)}$ .

**Question 6**

- (a) Find the eigenvalues and the eigenvectors of the matrix  $A = \begin{bmatrix} 1 & 2 \\ -2 & -2 \end{bmatrix}$ .
- (b) Solve the linear system  $4x + 2y - z = 5$ ,  $3x - y - z = -2$ ,  $x + y - z = 0$ .

*Good luck*

*Dr. Mona Mehanna*

*Dr. Mohamed Eid*

Answer the following questions:

[1] Complete the following statements:

(a) The circle is the locus of moving point such that....

(b) The equation  $x^2 - y^2 = 0$  represents pair of lines passing through .....and the angle between them is ....

[2] Separate the lines  $x^2 - 3xy + 2y^2 + 2x - 4y = 0$

[3] Write the equation of circle where the points  $(2, -1)$ ,  $(4, 1)$  are ends of diameter.

Also, determine its center and radius.

[4] Find the vertex and focus of the parabola  $y^2 - 4x - 4y + 16 = 0$

[5] Find the center, vertices and foci of the ellipse  $4x^2 + 9y^2 - 24x - 54y + 81 = 0$

Also, sketch its curve.

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*Good luck*

*Dr. Mohamed Eid*